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Screening versus confinement

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The influence of vacuum polarization on a confinement picture is discussed. The pair-production effects on the potential between two external charges are analyzed in the Schwinger model. Depending on the order of magnitude of the fermion mass, one obtains two clearly distinguished behaviors for the potential reflecting screening or confinement. In exceptional situations one finds colorless "quarks."

I. INTRODUCTION

There is an increasing consensus that quark confinement will be an essential ingredient of any future theory of strong interactions. Theoretical work strongly suggests that quantum chromodynamics (QCD) does exhibit confining properties.[1,2] One possible way of understanding this is via the formation of flux tubes by a mechanism dual to that known to occur in magnetic confinement.[2] The standard test for the existence of the linear potential implied by those flux tubes[3] is the Wilson loop criterion which usually is applied to the pure gauge theory, quarks being considered as external probes. One expects that if the quarks are very heavy, freezing the vacuum in this way will not affect the confining features of the theory, since in this case there should exist a critical length on the order of hadronic sizes below which pair production of quarks is negligible. For sufficiently large distances, however, polarization effects will take over and destroy the rising-potential picture. To get a more detailed understanding of what could be the influence of polarization on the confining properties of the potential, it is instructive to consider a gauge theory in two-dimensional space-time where the flux-tube formation is kinematic. As a consequence, the absence of "colored" states is also automatic in two-dimensional space-time. Indeed, consider a dipole state

$$|D\rangle = \psi(\vec{x}) P \exp\left[ig\int_{\vec{x}}^{\vec{y}} d\vec{z}\cdot\vec{A}(\vec{z})\right]\psi^*(\vec{y})\,|0\rangle\,, \qquad (1.1)$$

where $\vec{A}$ is the usual vector-potential matrix and P denotes path ordering. Since in the temporal gauge the electric field $\vec{E}$ is the canonical momentum conjugate to $\vec{A}$, one notices that the expectation value of the Hamiltonian grows linearly with the separation of the two quarks. This means that one cannot isolate particles carrying color.

Of course, a dipole state of the form (1.1) will have a growing energy, independent of the space-time dimensionality. However, in higher dimensions the real dynamical problem, related to flux-tube formation, is to find out whether this is the lowest-energy state compatible with Gauss's law. By staying in two-dimensional space-time, we avoid this dynamical problem and can concentrate on the effects of vacuum polarization on the confining picture.

For massive quarks we expect a linear rising potential up to distances of the order of $L = M/g^2$ where g is the coupling of the gauge field to the quarks. For very heavy quarks this should lead to the usual confinement picture where the observable colorless particles will manifest themselves as bound states formed out of a well-defined number of quarks. For vanishing mass on the other hand, color screening is expected to set in immediately,[4] since here no energy is required to create quark pairs, and the physical states are rather collective excitations analogous to plasmons. Thus, the absence of colored states can have rather different origins. In Sec. II we will illustrate these mechanisms using the Schwinger model as an example.

II. MODEL COMPUTATION

In this section we want to compute the potential between two static charges Q and $-Q$ separated by a distance $2L$ in the Schwinger model.[5] The corresponding Hamiltonian in boson-transformed form is given by[6]

$$H = \tfrac{1}{2}\int dx^1 \left\{\dot{\Sigma}^2 + (\nabla\Sigma)^2 + \mu^2(\Sigma-\phi)^2 + \frac{M^2}{2\pi}\left[1-\cos(2\sqrt{\pi}\Sigma)\right]\right\}, \qquad (2.1)$$

where $\mu = e/\sqrt{\pi}$ and

$$\phi(x^1) = \sqrt{\pi}\frac{Q}{e}\theta(x^1 + L)\theta(L - x^1).$$

The Hamiltonian (2.1) corresponds to the massive Schwinger model for vanishing chiral angle (background field of Ref. 7). The potential between those external charges is given by

$$H(L) = \min_{\psi} \langle\psi|H|\psi\rangle. \qquad (2.2)$$

The behavior of the potential for large L was found in Ref. 6 to approach a constant for values of Q which are integer multiples of e. This is clearly an effect of the dominating role of vacuum polarization for large separations. Here we wish to obtain a more detailed picture of the behavior of $H(L)$ for all L's. Since we do not hope to solve the quantum-mechanical problem exactly we will approximate it by the following classical one:

$$H(L) = \min \int dt\left[\frac{1}{2}\left(\frac{dq}{dt}\right)^2 - V_L(q,t)\right], \qquad (2.3)$$

where we have replaced x_1 by t and Σ by q for obvious reasons, and

$$V(q,t) = -\tfrac{1}{2}\mu^2[q - \phi(t)]^2 - \frac{M^2}{4\pi}[1 - \cos(2\sqrt{\pi}q)]. \qquad (2.4)$$

To obtain $H(L)$, Eq. (2.3), we have to consider the motion of a particle in the potential (2.4) subject to the boundary condition that for $t \to \pm\infty$ it is localized at $q = 0$ in order to ensure that the interparticle potential $H(L)$ be finite. During its motion, the particle changes at $t = -L$ from the potential V_0 to a new potential V_Q where the potentials are given by

$$V_Q(q) = -\tfrac{1}{2}\mu^2\left(q - \sqrt{\pi}\frac{Q}{e}\right)^2 - \frac{M^2}{4\pi}[1 - \cos(2\sqrt{\pi}q)],$$
$$V_0(q) = V_Q(q)\big|_{Q=0}, \qquad (2.5)$$

and reverses its motion at $t = 0$.

In computing $H(L)$ we find it convenient to make use of the relation

$$\frac{dH(L)}{dL} = -2\epsilon(L) = -2\mu q(-L)Q + Q^2, \qquad (2.6)$$

where $\epsilon(L)$ is the energy of the particle when it moves on the potential V_Q. Equation (2.6) follows from the minimal action principle since the boundary conditions are L independent.

Although $H(L)$ is trivially calculable in the massless Schwinger model ($M = 0$), we find it useful to first illustrate the procedure for obtaining $H(L)$ in this case.

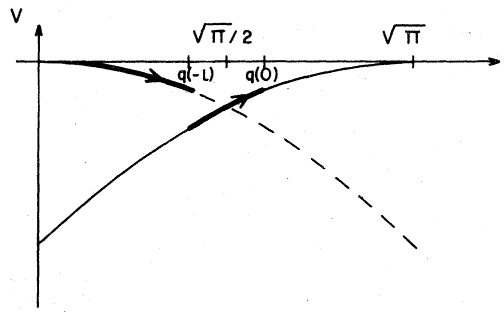

FIG. 1. Potentials V_0 (dashed line) and V_Q (solid line) for $M = 0$. Heavier line with arrow represents particle motion.

(*a*) *Case* $M = 0$. The potentials for this case are represented in Fig. 1. At time $t = -\infty$ the particle starts rolling down the potential hill V_0, moving from the origin to the right with zero total energy. At time $t = -L$ it is suddenly subjected to the new potential V_Q moving now with energy $\epsilon(L)$. The total time it spends on potential V_Q is given by

$$2L = 2\int_{q(-L)}^{q(0)} dq' \frac{1}{\{2[\epsilon - V_Q(q')]\}^{1/2}}. \qquad (2.7)$$

From (2.7) and (2.6) we obtain the well-known result[8]

$$H(L) = \frac{Q^2}{2\mu}(1 - e^{-2\mu L}). \qquad (2.8)$$

$H(L)$ can in no way be regarded as a confining potential, but rather corresponds to the immediate onset of the screening process. This total screening is seen to occur for any value of Q.

(*b*) *Case* $M \neq 0$. An inspection of the potentials

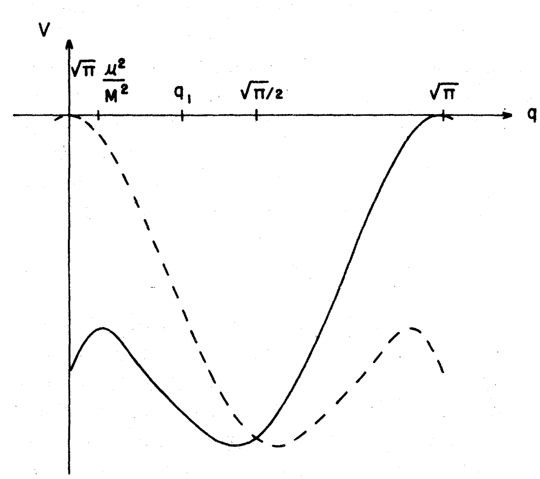

FIG. 2. Potentials V_0 (dashed line) and V_0 (solid line) for $M > e$, $\theta = 0$.

V_0 and V_Q shows that in this case $H(L)$ approaches a finite value as $L\to\infty$ only if the charge Q is quantized ($Q=ne$) in agreement with Ref. 6. Note that the growth of $H(L)$ for nonquantized charges is not to be taken as an indication for confinement since we are interested in probing the system with physical charges. In what follows we shall, therefore, consider only the case of integral charges, in particular $Q=e$, and $M\gg\mu$, where one does expect confining features.

The qualitative behavior of the corresponding potentials (for $M^2/\mu^2\simeq 10$) is depected in Fig. 2. There exist three classes of solutions satisfying the correct boundary conditions. For the first class (I), the particle moves between the origin and a turning point which approaches for $L\to\infty$ the local maximum of V_Q. This corresponds to $0\le L<\infty$ and gives rise to a linearly rising branch of $H(L)$ with $0\le L\le\infty$:

$$H(L)=\left(1-\frac{\mu^2}{M^2}\right)\pi\mu^2L+\frac{\pi\mu^4}{2M^3}(1-e^{-2ML}). \qquad (2.9)$$

The second and third class (II, III) of solutions only exist for $(1/M)\ln(M^2/\mu^2)\lesssim L<\infty$, with the "jumping point" $q(-L)$ lying in the ranges $[\sqrt{\pi}\mu^2/2M^2, q_1]$ and $[q_1,\sqrt{\pi}/2]$, respectively.

From Eq. (2.6) and the above characterization of classes II and III it is clear that the energy $H(L)$ corresponding to class II is higher than that of class III. For the solutions of class III, $H(L)$ is of the order of M (roughly the mass of a soliton) and up to corrections of order e/M is given by

$$H(L)=\frac{4M}{\pi}(1-2e^{-2ML}). \qquad (2.10)$$

It is clear that the true potential $H(L)$ is obtained by going smoothly from the branch I to the branch III (see Fig. 3) at the intersection point $L\simeq 4M/\pi^2\mu^2$, thus guaranteeing that the interparticle po-

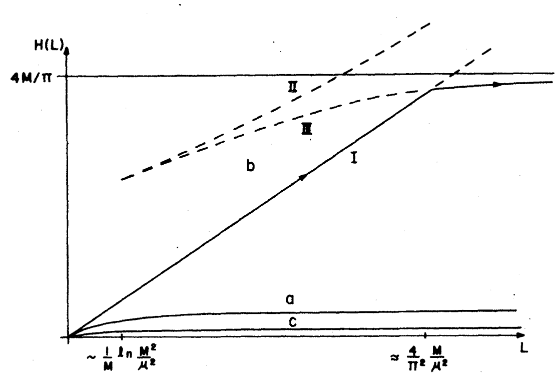

FIG. 3. Interparticle potential $H(L)$ in the three cases (a), (b), and (c). Dashed lines represent metastable states of the polarization cloud.

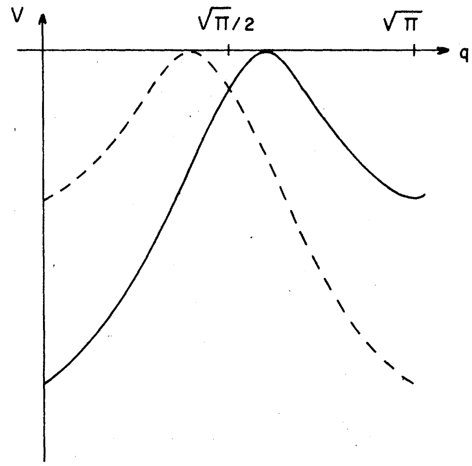

FIG. 4. Potentials V_0 (dashed line) and V_Q (solid line) for $M>e$, $\theta=\pi$.

tential has the smallest possible value for every L. As clearly seen from Fig. 3, for $M\gg\mu$ there exists a rather large region within which the potential is linearly rising. In the semiclassical approximation, this confining region is sharply distinguished from the domain in which pair production starts to become important.

(*c*) *Case* $\theta\ne 0$. So far, we considered the massive Schwinger model for the chiral angle $\theta=0$. The general case corresponds to shifting the argument of the cosine by a constant angle θ. One finds that for all $\theta\ne\pi$, the results closely parallel the preceding ones. For $\theta=\pi$, because of spontaneous symmetry breakdown (for $M\gg\mu$), kink states make their appearance, and one expects a breakdown of the confining picture. Indeed, as clearly seen from Fig. 4, there is in this case an additional degeneracy between the maxima of the two potentials.

Since the maxima are separated by a distance of only $\sqrt{\pi}\mu^2/M^2$, the motions in the equivalent mechanical problem minimizing the interparticle potential $H(L)$ will take place between these two maxima. In this case, just as in the massless Schwinger model, one finds

$$H(L)\simeq\frac{\pi}{2}\frac{\mu^4}{M^3}(1-e^{-2ML}). \qquad (2.11)$$

Comparing the above expression with the previous ones, one sees that not only is there no confining potential, but that screening effects are more violent than in the massless case. This leads us to interpret the above-mentioned kink states as colorless "quarks." Note that in contradistinction to the Coulomb potential of Ref. 7 which varies smoothly with θ, our $H(L)$ clearly distinguishes $\theta=\pi$ from all the other angles.

III. CONCLUDING REMARKS

Our model computations clearly show that an absence of "colored" states cannot necessarily be interpreted as a result of confinement in the usual sense. The confining theory should correspond to the situation found in case (b) where we have a potential which increases linearly up to distances of the order of $L_c \approx M/e^2$, whereas the size of a "quark-antiquark" bound state can be estimated to be of the order $(e/M)^{4/3}L_c$ which for $M \gg e$ is much smaller than L_c. In such a case we can understand a neutralization of "color" via the formation of bound states (hadrons). It is clear that the parameter M should be interpreted as being of the order of the "quark" mass, since for large M/e it sets the scale of the hadronic masses. For distances larger than the critical length, polarization effects become important and the whole potential picture breaks down through hadron formation. This is to be contrasted with the situation for the massless Schwinger model where we do not expect such a potential bound-state picture to hold and where "color" neutralization occurs via screening. It is interesting to note that even for heavy quarks, if $\theta = \pi$, one does not have a confining picture but rather a screening one. In this situation we are led to interpret the kink states as colorless quarks. This interpretation is supported by the fact that by coupling flavor to the Schwinger model one finds kink states which carry all the quantum numbers of the quarks except for color.[7,9,10] If the appearance of such colorless quarks is not a two-dimensional pathology, then one might have to worry about the possibility that vacuum polarization effects play an important role also in four-dimensional QCD.

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